RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2018

FIRST YEAR (BATCH 2018-21)

Date : 14/12/2018 **PHYSICS (Honours)**

Time : 11.00 am – 3.00 pm Paper : I Full Marks : 100

[Use a separate Answer Book for <u>each group</u>]

Group – A

(*Answer <u>any four</u> questions*) [4×10]

1. a) Is there a differentiable vector function $\vec{V}$ such that (a) $\vec{\nabla}\times\vec{V}=\vec{r}$ (b) $\vec{\nabla}\times\vec{V}=2\hat{i}+\hat{j}+3k$? if so, find $\vec{V}$. [1+5]

   b) If r is the distance of a point (x,y,z) from the origin then prove that $\vec{\nabla}\times\left(\hat{k}\times\vec{\nabla}\left(\frac{1}{r}\right)\right)+\vec{\nabla}\left(\hat{k}.\vec{\nabla}\left(\frac{1}{r}\right)\right)=0$ [4]

2. a) For any closed surface prove that— [3]

   $\oint_s \left[x(y-z)\hat{i}+y(z-x)\hat{j}+z(x-y)k\right]\bullet\vec{ds}=0$

   b) Suppose $\vec{A}=6z\hat{i}+(2x+y)\hat{j}-xk$. Evaluate $\iint_s \vec{A}.\vec{ds}$ over the entire surface S of the region bounded by the cylinder $x^2+z^2=9$, x = 0, y = 0, z = 0 and y = 8. [5]

   c) Find the volume of the region common to the intersecting cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$. [2]

3. a) Prove that a spherical polar co-ordinates system is orthogonal. [2.5]

   b) Represent a vector $\vec{A}=z\hat{i}-2x\hat{j}+yk$ in spherical polar coordinates and then determine A_r, A_θ and A_ϕ. [2.5]

   c) Express $\vec{\nabla}\times\vec{A}$ in orthogonal curvilinear co-ordinates system. [5]

4. a) Show that the solution of the equation $\frac{dy}{dx}+\frac{1}{x}y=0$, using ordinary power series gives trivial solution, why does this happen? Justify your answer. [2]

   b) Consider the equation $\frac{d^2u}{dx^2}+a\frac{du}{dx}+bu=0;$ a, b are constant coefficients.

   (i) Find the roots of the corresponding characteristic equation.

   (ii) If $(a^2-4b)=0$ is maintained then solve for u(x).

   (iii) Why the second solution only takes the form xu(x)? Why it does not take the other form like f(x)u(x)? Justify your answer. [1+1+2]

   c) $\frac{dy}{dx}=x-y;\quad y(1)=-1.$ Using power series seen at x=1 find the solution. [4]

5. a) Solve the differential equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod without radiation, subject to the following conditions.

   i) u is not infinite for $t\to\infty$

   ii) $\frac{\partial u}{\partial x}=0$ for x=0 and x = l

iii) $u = lx - x^2$ for t=0 , between x=0 and x=l [7]

b) Solve $\left(y^2e^{xy^2} + 4x^3\right)dx + (2xye^{xy^2} - 3y^2)dy = 0$ [3]

6. Legendre equation is written as $\left(1-x^2\right)y'' - 2xy' + l(l+1)y = 0.$ Use Frobenius ascending power series $\left[y(x) = \sum_{n=0}^{\infty} a_n x^{n+r}\right]$ substitution to give the answer of the following question.

a) Verify that the indicial equation is $r(r-1) = 0$ [2]

b) Using r=0 , obtain a series of even powers of x, i.e. y_{even} (considering a_1=0) [2]

c) Using r=1, develop a series of odd powers of x, i.e. y_{odd} (considering a_1=0). [2]

d) Show that both solutions, y_{even} and $y_{odd,}$ diverge for $x = \pm 1$ if the series continue to infinity. [2]

e) Finally show that by an appropriate choice of l, one series at a time may be converted into a polynomial, thereby avoiding the problem related with divergence. [2]

7. a) Can f(x)=tan x be expanded in a Fourier series? Explain. [2]

b) Find the Fourier transform of the function $f(x) = e^{-\alpha x^2}$ (α>0, constant) [3]

c) Show that, Fourier integral can be written in terms of delta function. [2]

d) Show that $\int_{-\infty}^{\infty} dx\, e^{-|x|}\, \delta(\sin x) = \coth\frac{\pi}{2}$ Hint: $\delta(g(x)) = \sum_i \frac{\delta(x - x_i)}{\left|\frac{dg(x)}{dx}\right|_{x=xi}}$, where the sum runs over all the real zeros of g(x). [3]

Group – B

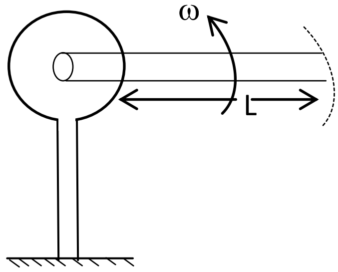
(*Answer <u>any three</u> questions*) [3×10]

8. a) A particle moves in plane polar coordinates with r = 2a cosθ & $\dot{\theta} = h/r^2$. Find the velocity, acceleration and hence the force acting on the particle. [1+2+1]

b) Show that the motion of the projectile as seen from another projectile will always be straight line motion. [3]

c) A wheel has eight equally spaced spokes and a radius of 30 cm. It is mounted on a fixed axle and is spinning at 2.5 rev/s. You want to shoot a 20 cm long arrow parallel to this axle and through the wheel without heating any of the spokes. Assume that the arrow and the spokes are very thin. (a) What minimum speed must the arrow have? (b) Does it matter where between the axle and rim of the wheel you aim? If so, what is the best location? [2+1]

9. A uniform rope of mass M and length L is pivoted at one end and whirls in a horizontal plane with constant angular speed ω. To find the tension in the string answer the following question. Neglect gravity.



a) Find the equation of motion for a small section of rope between r and r+Δr.
Hint: Because the motion is circular, the section is undergoing radial acceleration.
This requires a net radial force, which is possible only if the forces pulling the ends of the section are not equal. [3]

b) Find the tension as a function of r. Consider T_o, T(r) is the tension at r = 0 and at r respectively. [4]

c) Evaluate T_o

Hint: T(L) = 0 because the end of the rope at r = L is free. [2]

d) Show that $T(r)=\frac{M\omega^2}{2L}(L^2-r^2)$ [1]

10. a) A particle moves in a circular orbit under the action of a force $f(r)=-\frac{k}{r^2}$. If k is suddenly reduced to half its original value, show that the particle would move along a parabola. [3]

b) (i) Prove Kepler's third law using the fact that for an ellipse semi-minor axis b is equal to $\sqrt{1-\varepsilon^2}$ times the semi-major axis, where $\varepsilon=\sqrt{1+\frac{2EL^2}{Mk^2}}$ is the eccentricity for inverse square law of force, where the symbols have their usual meanings.

(ii) Before landing man on the moon, the Apollo 11 space vehicle was put into orbit about the moon. The mass of the vehicle was 9979 kg and the period of orbit was 119 min. The maximum and minimum distance from the centre of the moon were 1861 km and 1838 km. Assuming the moon to be a uniform spherical body, what is the mass of moon according to these data? [2+2]

c) Show that in Rutherford scattering the trajectory of the particle will be a hyperbolic. [3]

11. a) Determine the gravitational field and gravitational potential at any point (i) inside and (ii) out side the surface of the Earth. Assume that the Earth is of solid sphere of uniform density. [4]

b) Show that the gravitational potential inside a thin spherical shell is constant. [3]

c) A solid cylindrical rod has half the length as a hollow cylindrical rod (of the same material and same mass) which has an external radius $\sqrt{2}$ times the internal radius. Show that the torsional rigidities of the two rods are in the ratio of 8:3. [3]

12. a) A mass m moves along a straight line under the influence of a constant force F. Assuming that there is a resesting force. numerically equal to kv^2 where ν is the instantaneous speed and K is a constant, prove that the distance travelled in going from speed v_1 to v_2 is

$$\frac{m}{2k}\ln\left(\frac{F-kv_1^2}{F-kv_2^2}\right).$$ [3]

b) Derive an expression for the equation of continuity of an ideal fluid of density ρ. What is the form of this equation, when the fluid is incompressible? [3+1]

c) Two vessels of equal cross-section α, are joined near their bases by a horizontal narrow tube of length *l* and internal radius r. Initially the liquid surfaces are at heights 3h and h respectively above the capillary tube. Calculate the time taken for the difference in levels to become h, if the coefficient of viscosity is η and density ρ. The flow is assumed to be slow. [3]

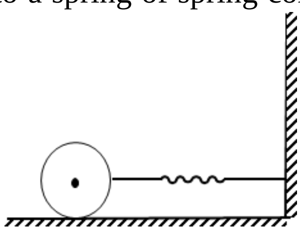
<u>Group – C</u>

(*Answer <u>any three</u> questions*) [3×10]

13. a) Staring from the conservation of energy, find the expression for time period of a simple pendulum. [3]

b) Given a differential equation: $\frac{d^2y}{dx^2}+\omega^2y^2=0$, Let y_1,y_2 be the two independent solution of the above equation. Show that linear superposition of those solutions will not satisfy the above equation. Comment on the nature of differential equation. [3]

c) A circular solid cylinder of radius 'r' and mass 'm' is connected to a spring of spring constant 'k' as shown in figure. Determine the frequency of horizontal oscillations of the system if the cylinder,



i) slips on the surface without rolling.

ii) rolls on the surface without slipping.(Neglect friction) [4]

14. a) Show that the average energy of a weakly damped harmonic oscillator decay exponentially with time. [3]

b) A massless spring, suspended from a rigid support, carries a flat disc of mas 100 g at its lower end. It is observed that the system oscillates with a frequency of 10 Hz and the amplitude of the damped oscillations reduces to half its undammed value is one minute. Calculate:

i) the resistive force constant,

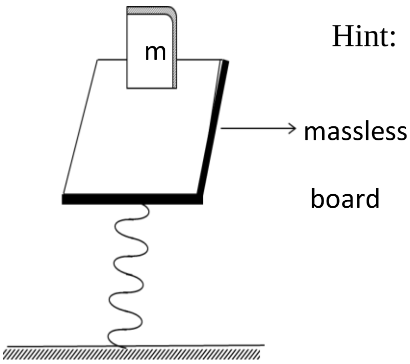
ii) the relaxation time of the system,

iii) its gravity factor

iv) the force constant of the spring. [4]

c) What is 'logarithmic decrement'? How is it determined? Illustrate with one example. [3]

15. A vertical spring of force constant k and natural length *l* is fixed at one end to a horizontal table. At the other end of the spring is attached a massless board on which a 100 gm mass is kept in static equilibrium. A vertical force $F=F_o\sin(2\omega_o t)$ is applied to a board, where $\omega_o = \sqrt{k/m}$.What is the maximum value of F_0 for which the mass remains in contact with the board? [1+2+3+4]



Hint: (i) Consider the forces acting on mass m and write equation of motion.

(ii) Determine the reaction force N exerted by the board on m.

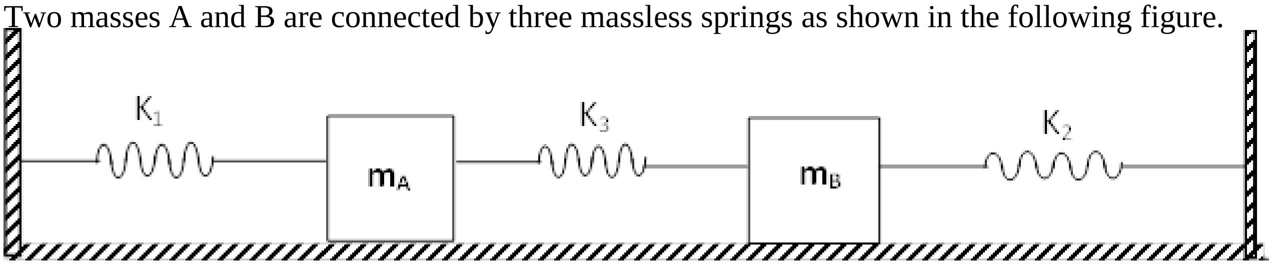
It can be obtained from the equation of motion of the board.

(iii) Find the distance of the board as measured from the static equilibrium point

(iv) For the mass to be in contact i.e. N > 0. Estimate that $F_o < 0.537$ N

16. a) Draw in figure to show the normal modes of CO_2. [3]

b) Two masses A and B are connected by three massless springs as shown in the following figure.



If $m_A=m_B = m$ are $K_3 = \sqrt{K_1K_2}$ then,

i) find the frequencies of the normal modes for longitudinal oscillations.

ii) Find the normal coordinates of the system. [4]

c) Trace graphically and analytically the motion of a particle that is subjected to two perpendicular simple harmonic motion of equal frequencies, different amplitude and phases differing by i) zero ii) $\pi/2$. [3]

17. a) Derive the differential equation of motion for transverse vibrations of a uniform flexible stretched string. [4]

b) A plane wave has amplitude 0.001 cm, frequency 200 H_Z and wavelength .150 cm.

i) Write down an expression for the wave.

ii) Calculate its phase velocity and phase difference between two points distance 30 cm apart along the line of propagation. [3]

c) What is 'Standing wave'? Obtain the expression for 'Doppler effect' if both the source and the observer are in motion. [3]

———— × ————